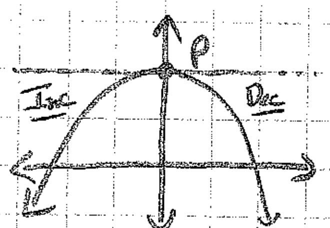
Unit 11 - Graphing with Derivatives

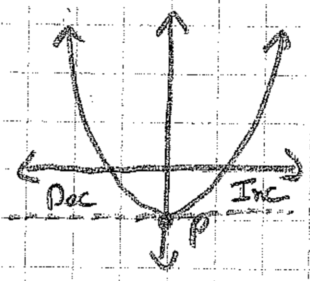
Day #1 - Sketching Polynomial Functions

<u>Objective</u> - You will sketch polynomial functions by using the first + second derivative.

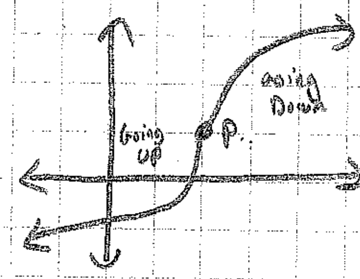
<u>Absolute Max:</u>

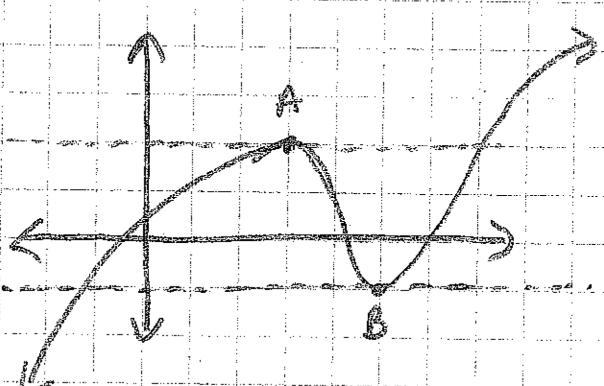


<u>Absolute Min:</u>



<u>Point of Inflection</u>

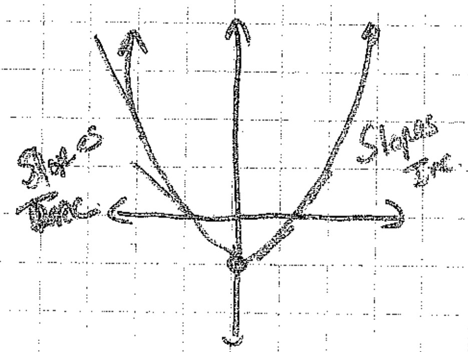




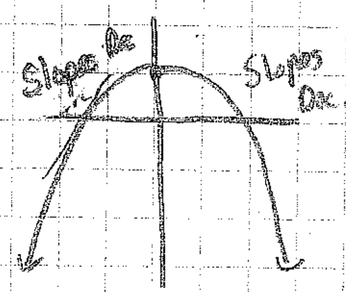
Local Max or Relative Max - Max Value on same interval.

Local Min or Relative Min - Min Value on same interval.

<u>Concave Up</u>



<u>Concave Down</u>



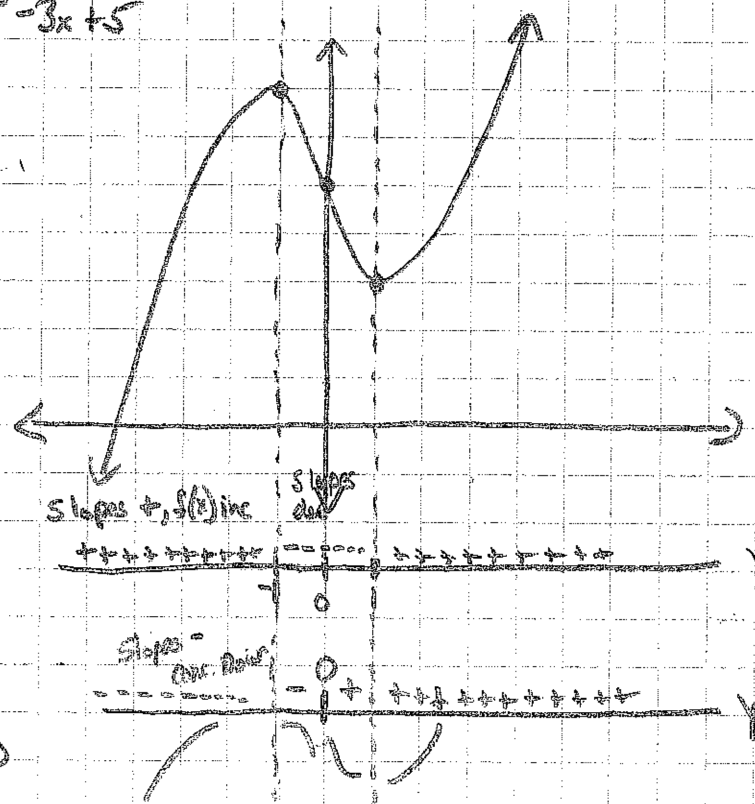
To find max/min points:

1) Find the first derivative
2) Set the derivative equal to 0.
3) Solve for x
4) Find the y-values by substituting x into the original function.

To find the points of inflection:

1) Find the second derivative.
2) Follow steps 2-4 from max/min.

Example #1: **Sketch:** $y = x^3 - 3x + 5$



x-int: $(y=0)$

$0 = x^3 - 3x + 5$ (Not happening)

y-int $(x=0)$:

$y = 0^3 - 3(0) + 5$

$y = 0 - 0 + 5$

$y = 5$

$(0, 5)$

Critical Points (Max/Min)

$y' = 3x^2 - 3$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0 \longrightarrow 3(-)(-1)$

$3+$

$3 \neq 0$ $\quad x+1=0 \quad \boxed{x=-1}$ $\quad x-1=0 \quad \boxed{x=1}$

$y = (-1)^3 - 3(-1) + 5$

$y = -1 + 3 + 5$

$\boxed{y = 7}$

$(-1, 7)$

$y = (1)^3 - 3(1) + 5$

$y = 1 - 3 + 5$

$\boxed{y = 3}$

$(1, 3)$

POI:

$y'' = 6x$

$\frac{6x}{6} = \frac{0}{6}$

$\boxed{x = 0}$

$y = 0^3 - 3(0) + 5$

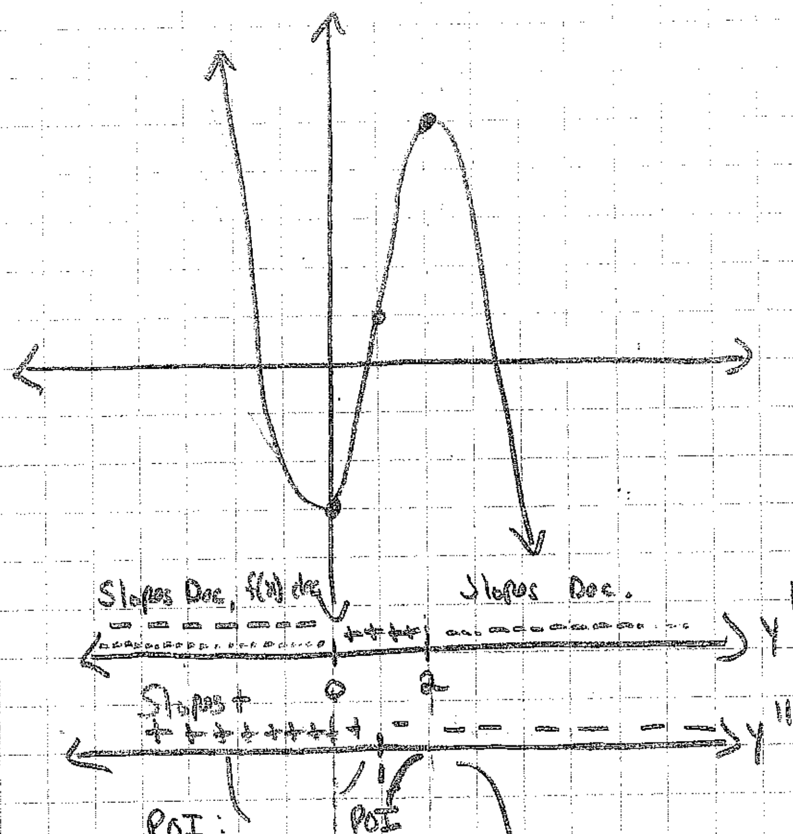
$y = 0 - 0 + 5$

$\boxed{y = 5}$

$(0, 5)$

Example #2 Sketch: $y=-2x^3+6x^2-3$

Xaint: $(y=0)$
$0=-2x^3+6x^2-3$
(no!)



y-int: $(x=0)$
$y=-2(0)^3+6(0)^2-3$
$y=0+0-3$
$y=-3$
$(0,-3)$

Critical Pts (Max/Min)

$y'=-6x^2+12x$
$-6x^2+12x=0$ $\quad -6(+)(+)$
$-6x(x-2)=0$ $\quad -(-)(-)$
$\frac{-6x}{-6}=\frac{0}{-6}$ $\quad x-2=0$ $\quad -6(1)(1-2)$
$\boxed{x=0}$ $\quad \boxed{x=2}$

$y=-2(0)+6(0)^2-3$
$y=0+0-3$
$\boxed{y=-3}$
$(0,-3)$

$y=-2(2)^3+6(2)^2-3$
$y=5$
$(2,5)$

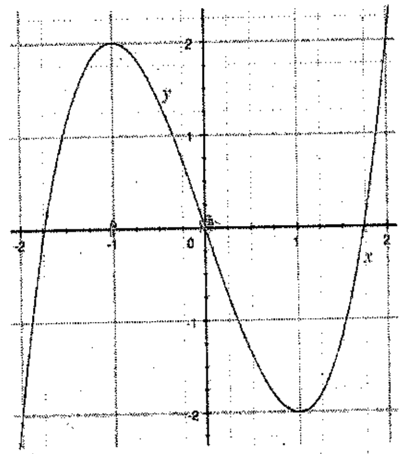
POI:
$y''=-12x+12$
$-12x+12=0$
$-12 \quad -12$
$\frac{-12x}{-12}=\frac{-12}{-12}$
$\boxed{x=1}$
$y=-2(1)^3+6(1)^2-3$
$y=-2+6-3$
$\boxed{y=1}$ $\quad \boxed{(1,1)}$

HW: Sketch 1) $y=3x^3-9x+5$
2) $y=x^3-6x^2+9x+1$

Unit 11 Day #2 - Interval Practice

Objective - You will describe the following intervals.

1) Determine the intervals the graph is:



a) Increasing: $(-\infty, -1)(1, \infty)$

Decreasing: $(-1, 1)$

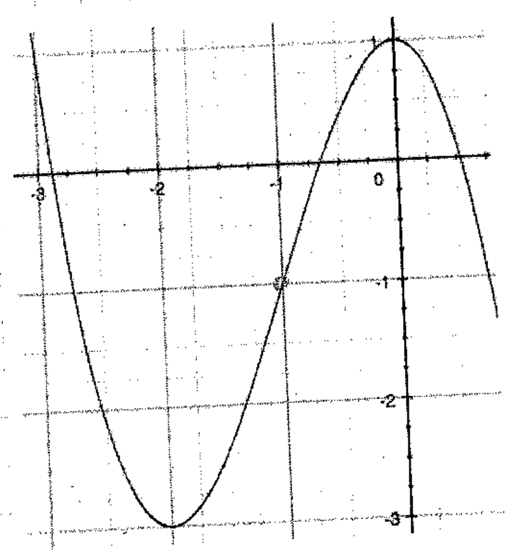
Concave Up: $(0, \infty)$

Concave Down: $(-\infty, 0)$

End Behavior:

As $x \to +\infty$, $f(x) \to +\infty$
As $x \to -\infty$, $f(x) \to -\infty$

2) Determine the intervals the graph is:



Increasing: $(-2, 0)$

Decreasing: $(-\infty, -2)(0, \infty)$

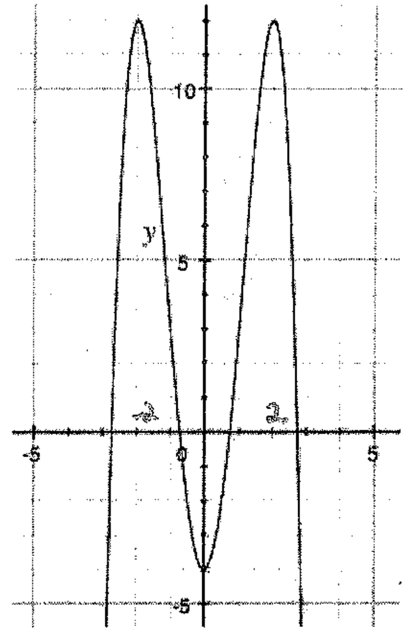
Concave Up: $(-\infty, -1)$

Concave Down: $(-1, \infty)$

End Behavior:

As $x \to +\infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to +\infty$

3) Determine the intervals the graph is:



<u>Increasing</u>: $(-\infty,-2)$ $(0,2)$

<u>Decreasing</u>: $(-2,0)$ $(2,\infty)$

<u>Concave Up</u>:

<u>Concave Down</u>:

<u>End Behavior</u>:

As $x \to +\infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

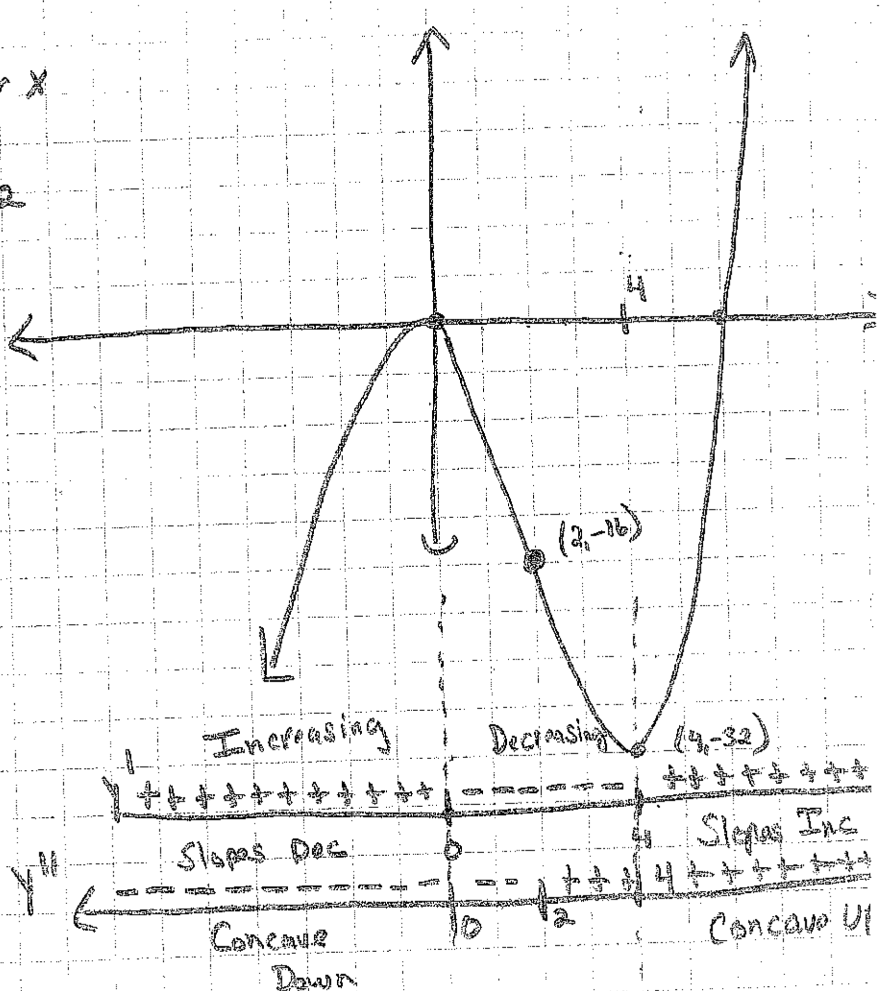
Unit 11 Day #3 - More Graphing - Find the x and y-intercepts

Objective - You will continue to graph functions

To find x-intercepts:
1) Substitute 0 for y
2) Solve for x

To find y-intercepts
1) Substitute 0 for x
2) Solve for y

Ex: **Given:** $y = x^3 - 6x^2$



x-int: $(y=0)$

$0 = x^3 - 6x^2$

$x^2(x-6) = 0$

$x^2 = 0$	$x - 6 = 0$
$\boxed{x = 0}$	$\boxed{x = 6}$
$(0,0)$	$(6,0)$

y-int: $(x=0)$

$y = 0^3 - 6(0)^2$

$y = 0 \quad (0,0)$

Critical Pts (Max/Min)

$y' = 3x^2 - 12x$

$3x(x-4) = 0$

$3x = 0 \qquad x - 4 = 0$

$\boxed{x = 0} \qquad \boxed{x = 4}$

$3(-)(-)$ +

$3(1)(1-4) = -$

$3(+)(+) = +$

$y = 0^3 - 6(0)^2 \qquad y = 4^3 - 6(4)^2$

$y = 0 \qquad y = -32$

$(0,0) \qquad (4,-32)$

POI

$y'' = 6x - 12 \qquad y = (2)^3 - 6(2)^2$

$6x - 12 = 0 \qquad \boxed{y = -16}$

$6x = 12$

$\boxed{x = 2} \qquad (2,-16)$

Increasing: $(-\infty, 0)\ (4, \infty)$

Decreasing: $(0, 4)$

Concave Up: $(2, \infty)$

Concave Down: $(-\infty, 2)$

End Behavior: As $x \to +\infty$, $f(x) \to$

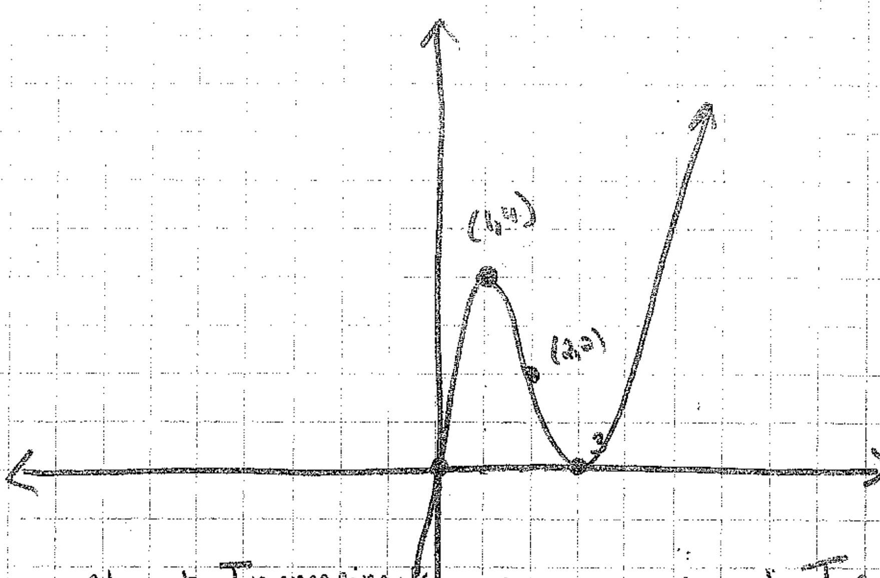
As $x \to -\infty$, $f(x) \to -$

2) Given: $y = x^3 - 6x^2 + 9x$

x-int (y=0)

$$x^3 - 6x^2 + 9x = 0$$
$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)(x-3) = 0$$

$x=0$	$x=3$	$x=3$
$\boxed{(0,0)}$	$\boxed{(3,0)}$	



y-int (x=0):

$$y = 0^3 - 6(0)^2 + 9(0)$$
$$y = 0 \quad (0,0)$$

Slopes + Increasing | Dec. | Slopes + Inc.

y': + + + + + + + + + + + + + + + | 1 – – – | 3 + + + + + + + + +

Slopes (–) Conc Down | Slopes In

y'': – – – – – – – – – – – – – | 1 – | 2 + | 3 + + + + + + + +

Concave up.

Critical Pts

$$y' = 3x^2 - 12x + 9$$

$$3x^2 - 12x + 9 = 0$$
$$3(x^2 - 4x + 3) = 0$$
$$3(x-3)(x-1) = 0$$

$3 \neq 0$	$x-3=0$	$x-1=0$
	$\boxed{x=3}$	$\boxed{x=1}$

$y = (3)^3 - 6(3)^2 + 9(3)$ $y = (1)^3 - 6(1)^2 + 9(1)$

$\boxed{y=0}$ $(3,0)$ $\boxed{y=4}$ $(1,4)$

Increasing: $(-\infty, 1)\ (3, +\infty)$

Decreasing: $(1, 3)$

Concave Up: $(2, \infty)$

Concave Down: $(-\infty, 2)$

POI:

$$y'' = 6x - 12$$
$$6x - 12 = 0$$
$$\frac{6x}{6} = \frac{12}{6}$$
$$\boxed{x = 2}$$

$$y = (2)^3 - 6(2)^2 + 9(2)$$
$$\boxed{y = 2}$$
$$(2,2)$$

End Behavior:

As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to -\infty$

HW: $y = 2x^3 - 6x^2$

Go!!

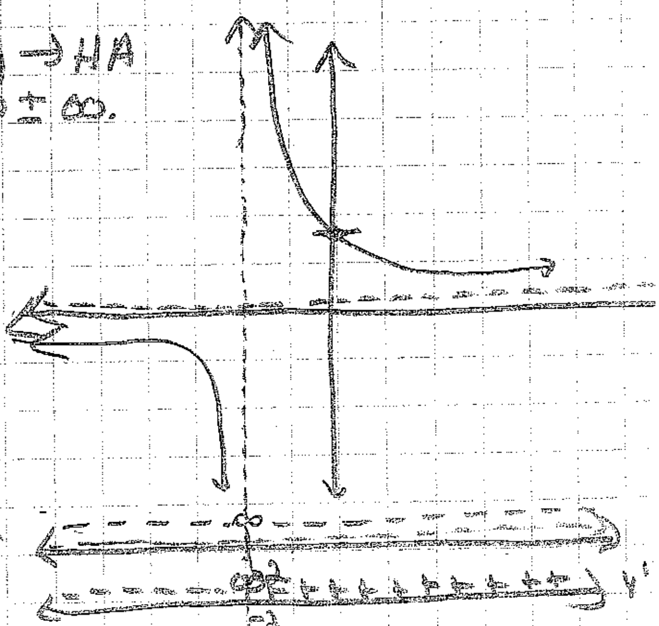
Unit 11 Day #4 – Graphing RATS (Rational Functions)

<u>Objective</u> – You will graph rational functions using the first and second derivates.

<u>Graphing RATS</u>

1) Determine x and y intercepts
2) Determine vertical asymptotes (where the graph is undefined – set den = 0)
3) Determine horizontal asymptotes (limit of the graph as $x \to \pm\infty$)
4) Determine if the graph crosses the HA (Plug answer from #3 into original and see what happens)
5) Using a number line, determine where the graph is positive and negative (Put x-int and VA on number line)
6) Graph
   - as $x \to \pm\infty$, $f(x) \to HA$
   - as $x \to VA$; $f(x) \to \pm\infty$.

<u>Graph</u>: $y = \frac{3}{x+2}$



<u>x-int</u> (y=0):

$0 = \frac{3}{x+2}$

$0 = 3$ $\boxed{NONE}$

<u>y-int</u>:

$y = \frac{3}{0+2}$

$\boxed{y = \frac{3}{2}}$

<u>VA</u>

$x + 2 = 0$

$\boxed{x = -2}$

<u>HA</u>

$\lim_{x \to \infty} f(x) \frac{3}{x+2} = 0$

$\boxed{y = 0}$

$f = 3$ $g = x+2$

$f' = 0$ $g' = 1$

<u>Crosses HA?</u>

If so $y = 0$

$\therefore$

$0 = \frac{3}{x+2}$

$0 = 3$

$\boxed{NO}$

<u>Critical Points</u>

$y' = \frac{g \cdot f' - f \cdot g'}{g^2}$

$y' = \frac{(x+2)(0) - 3(1)}{(x+2)^2}$

$y' = \frac{-3}{(x+2)^2}$ $0 = \frac{-3}{(x+2)^2}$ $-3 \neq 0$ $\boxed{NONE}$

<u>POI</u>

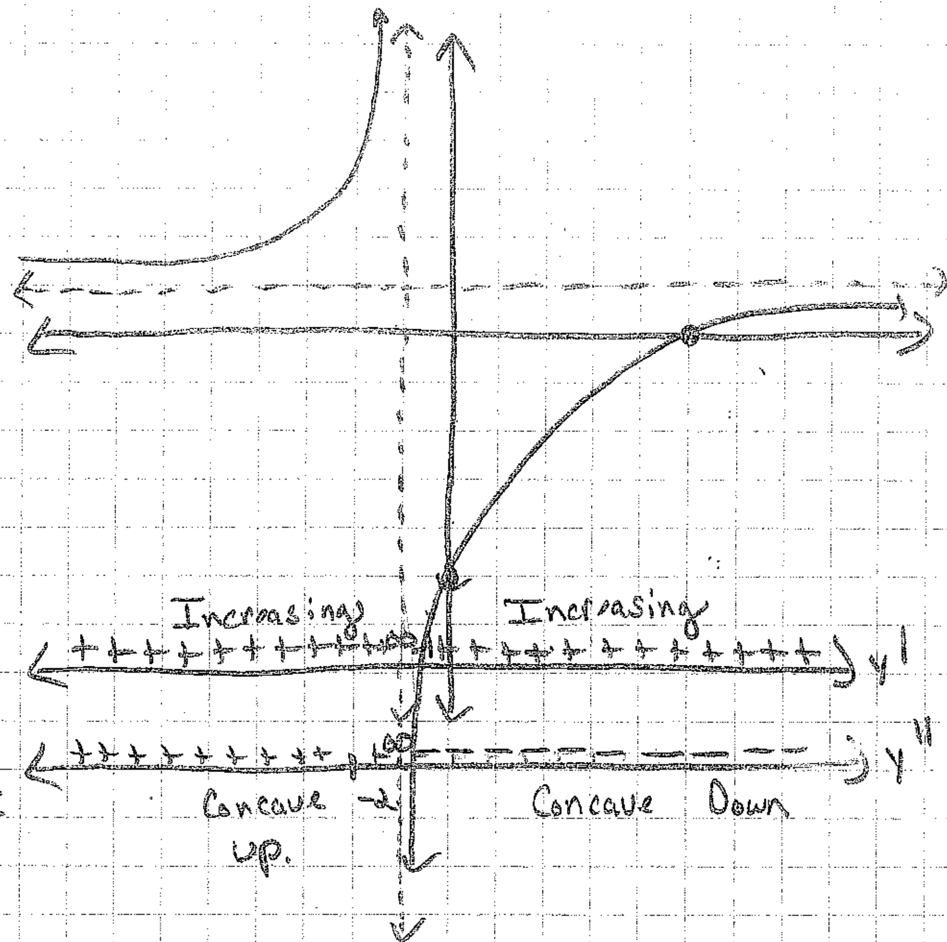
$y'' = \frac{(x+2)^2(0) - (-3)(2(x+2))}{((x+2)^2)^2}$

$y'' = \frac{6(x+2)}{(x+2)^4} = 0$

$6x + 12 = 0$

$\boxed{x = -2}$

2) Graph: $y = \dfrac{x-5}{x+1}$



x-int: (y=0)

$\dfrac{0}{1} = \dfrac{x-5}{x+1}$

$x-5=0$

$x=5$

$\boxed{(5,0)}$

y-int: (x=0):

$y = \dfrac{0-5}{0+1}$

$y = \dfrac{-5}{1}$

$\boxed{y=-5}$ $(0,-5)$

VA:

$x+1=0$

$\boxed{x=-1}$

HA:

$\lim\limits_{x\to\infty} f(x) = \dfrac{x-5}{x+1}$

$\boxed{=1}$

Crosses HA?

If so $y=1$: $1 = \dfrac{x+5}{x+1}$

$x+1 = x+5$

$\boxed{NO}$

Critical Pts:

$f = x-5 \quad g = x+1$

$f' = 1 \quad g' = 1$

$y' = \dfrac{g \cdot f' - f \cdot g'}{g^2}$

$y' = \dfrac{(x+1)(1) - (x-5)(1)}{(x+1)^2}$

$y' = \dfrac{x+1-(x-5)}{(x+1)^2}$

$y' = \dfrac{x+1-x+5}{(x+1)^2}$

$y' = \dfrac{6}{(x+1)^2}$

POI:

$y'' = \dfrac{g \cdot f' - f \cdot g'}{g^2}$

$f = 6 \quad g = (x+2)^2$

$f' = 0 \quad g' = 2(x+2)$

$y'' = \dfrac{(x+2)(0) - 6(2(x+2))}{((x+2)^2)^2}$

$y'' = \dfrac{0 - 12(x+2)}{(x+2)^4}$

$y'' = \dfrac{-12(x+2)}{(x+2)^4}$

$y'' = \dfrac{-12x-24}{(x+2)^4}$

$\dfrac{0}{1} = \dfrac{-12x-24}{(x+2)^4}$

$-12x-24=0$

$\dfrac{-12x}{-12} = \dfrac{24}{-12}$ $\boxed{x=-2}$

HW: Graph $\rightarrow y = \dfrac{-4}{x-1}$

Unit 11 Day #5 - Graphing RATS w/ Holes

Objective - You will graph RATS w/ holes.

Graphing RATS w/Holes

If a term containing an x can be reduced, the graph has a hole at that x-value.

Example #1

Graph: $y = \dfrac{-x^2}{x^3-4x} = \dfrac{-\cancel{x^2}}{x(x^2-4)} = \boxed{\dfrac{-x}{x^2-4}}$ Hole @ $\boxed{x=0}$

X int (y=0)

$0 = \dfrac{-x^2}{x^3-4x}$

$-x^2 = 0$

$\boxed{x=0}$

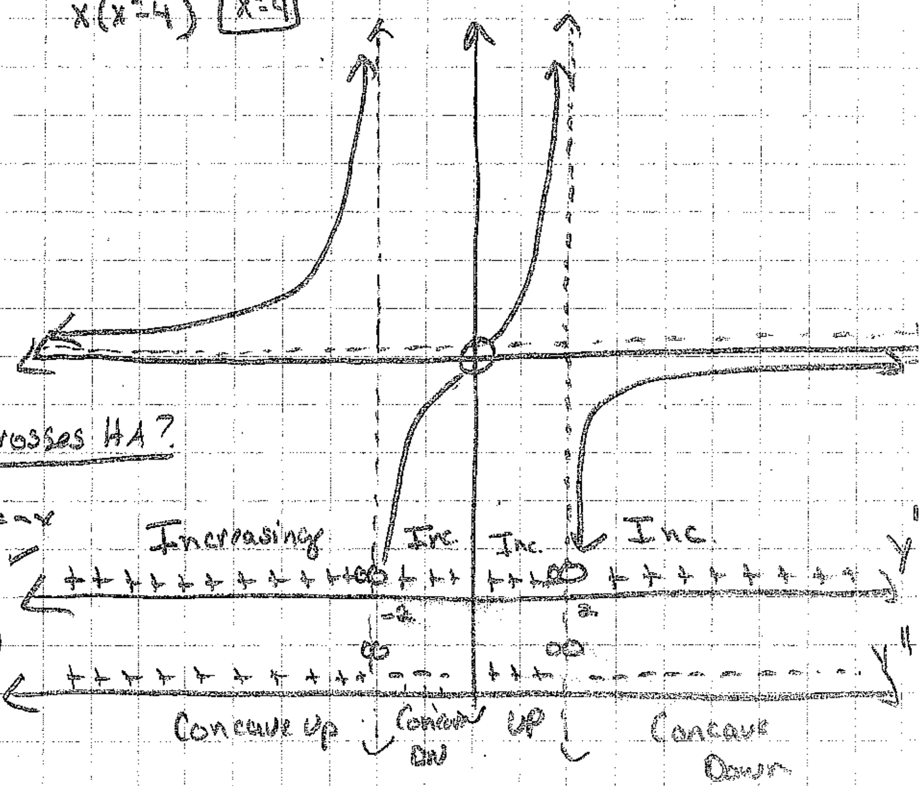
$(0,0)$

y int (x=0)

$y = \dfrac{-(0)^2}{0^3-4(0)}$

$y=0$

$(0,0)$



VA:

$x^2-4=0$

$x^2=4$

$\boxed{x=\pm 2}$

HA:

$\lim\limits_{x\to\infty} f(x)$

$\boxed{=0}$

Crosses HA?

$0=-x$

yes ✓

@ $\boxed{x=0}$

Max/Min

$f=-x$ $\quad g=x^2-4$ $\quad y' = \dfrac{g\cdot f' - f\cdot g'}{g^2}$

$f'=-1$ $\quad g'=2x$

$y' = \dfrac{(x^2-4)(-1)-(-x)(2x)}{(x^2-4)^2}$

$y' = \dfrac{-x^2+4+2x^2}{(x^2-4)^2}$

$y' = \dfrac{x^2+4}{(x^2-4)^2}$

$0 = \dfrac{x^2+4}{(x^2-4)^2}$ $\qquad x^2+4=0$

$\{\ \}$

no max/min

POI:

$f=x^2+4$ $\quad g=(x^2-4)^2$ $\quad y'' = \dfrac{g\cdot f' - f\cdot g'}{g^2} = \dfrac{(x^2-4)^2(2x)-(x^2+4)\,2(x^2-4)(2x)}{((x^2-4)^2)^2}$

$f'=2x$ $\quad g'=2(x^2-4)(2x)$

Give $\boxed{y'' = \dfrac{-2x^3-24x}{(x^2-4)^3} = 0}$

$-2x(x^2+12)=0$

$-2x=0$	$x^2+12=0$
$x=0$	$x^2=-12$
(hole)	$\{\ \}$

2) Graph: $y = \frac{x^2 - 2x}{x^2 + 3x - 10} = \frac{x(x-2)}{(x+5)(x-2)} = \frac{x}{x+5}$

$x - 2 = 0$
$x = 2$
Hole @ $x = 2$

x-int (y=0):

$\frac{0}{1} = \frac{x}{x+5}$

$\boxed{x = 0}$

$(0,0)$

y-int (x=0):

$y = \frac{0}{0+5}$

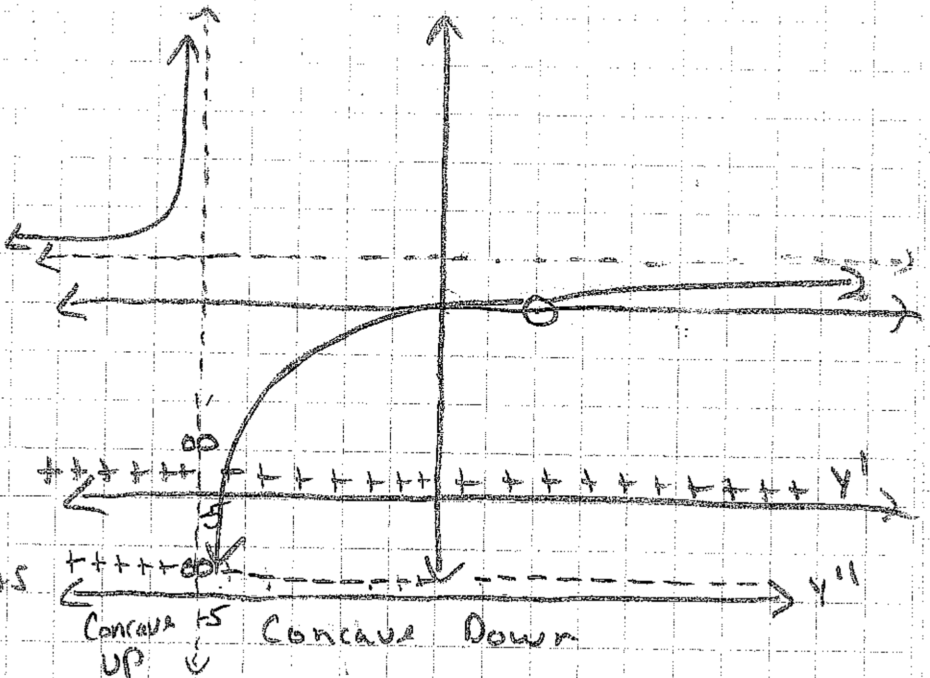
$\boxed{y = 0}$
$\boxed{(0,0)}$

VA:

$x + 5 = 0$
$\boxed{x = -5}$

VA:

$\lim_{x \to \infty} f(x)$
$\boxed{= 1}$



Max/min:

$f = x \quad g = x+5$
$f' = 1 \quad g' = 1$

$y' = \frac{g \cdot f' - f \cdot g'}{g^2}$

$y' = \frac{(x+5)(1) - x(1)}{(x+5)^2}$

$y' = \frac{x+5-x}{(x+5)^2}$

$y' = \frac{5}{(x+5)^2}$

$\frac{0}{1} = \frac{5}{(x+5)^2}$

$5 = 0$
NO MAX/MIN

POI:

$f = 5 \quad g = (x+5)^2$
$f' = 0 \quad g' = 2(x+5)$

$y'' = \frac{g \cdot f' - f \cdot g'}{g^2}$

$y'' = \frac{(x+5)^2 \cdot (0) - 5(2(x+5))}{((x+5)^2)^2}$

$y'' = \frac{0 - 10(x+5)}{(x+5)^4}$

$\frac{0}{1} = \frac{-10x-50}{(x+5)^4}$

$y'' = \frac{-10x - 50}{(x+5)^4}$

$-10x - 50 = 0$
$-10x = 50$
$\frac{-10x}{-10} = \frac{50}{-10}$ $\boxed{x = -5}$

HW: Graph:
1) $y = 3x^3 - 9x^2$
2) $y = \frac{4x}{9x - x^2}$